Construction of efficient models for turbomachinery with cracks using X-X_r and BAA

Undergraduate Thesis

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Abstract

Analysis of the influence of cracks on turbomachinery is important for design, failure prognosis and structural health monitoring. However, predicting the dynamics of turbomachinery with cracks is relatively challenging because the size of industrial turbomachinery models is typically quite large since they need to capture the complex geometry. The large model size combined with the nonlinearity due to the cracks in the turbomachinery requires a considerable computational effort. To deal with this difficulty, an efficient method to build reduced order models (ROMs) for turbomachinery with cracks is developed. This technique employs the relative coordinates to describe the motion of the crack surface and is referred to as X-Xr approach. The relative coordinates are defined as the relative displacements between contact pairs along the crack surface. The matrices of the governing equations are divided into the relative and pristine components. Due to the specialized form of the X-Xr approach, the ROM of the turbomachinery with a crack is constructed from models of the cracked and pristine sector models alone, without the need for constructing the full stage model of the system, and thus greatly lowering the computational expense. The bilinear amplitude approximation (BAA) is combined with the ROM to quickly estimate the forced response of the piecewise-linear nonlinear system using linear analysis. The X-Xr method has been used on a turbomachinery model with a crack to lower the size of the model from approximately 80,000 degrees of freedom to approximately 200 degrees of freedom. In addition to the ROM matching the modal properties of the full-order system, the forced response analysis from using the ROM with BAA has also been shown to match the full-order nonlinear result. The work presents a way to create a ROM that can be used to analyze turbomachinery with cracks at a fraction of the time that is needed for the full-order analysis.

Acknowledgments

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Chapter 1: Introduction

1.1 Background

Nowadays, with the development of industry, the nonlinearity like cracks will happen in the cyclic structure including the bladed disk during the operation. The reasons of causing the crack existing in the bladed disk will be a lot including the collision caused by foreign object and metal fatigue after excessive rotation. With the increase of time, the crack will extend until broken state. Figure 1 is the damage to bladed disk. Hence, there has been a growing interest in predicting the dynamics of cyclic structure with cracks to be better to account for the effect of crack existing in the bladed disk recently.

Figure 1: Damage to bladed disk [29]

Figure 2 is the geometry of bladed disk with a crack in one sector shown below. The model of bladed disk with cracks is composed by one cracked sector and other pristine sectors. Figure 3 is the geometry of single pristine sector and single cracked sector.

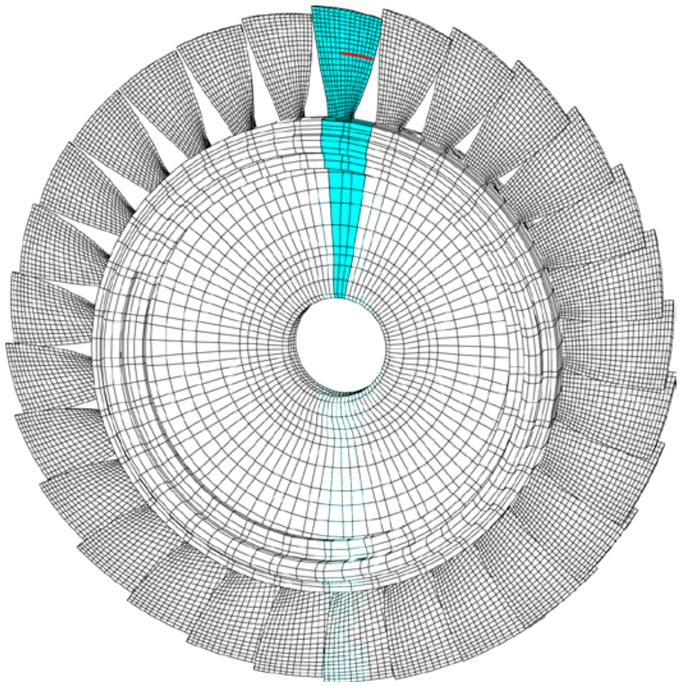

Figure 2: Bladed disks with a crack in one sector

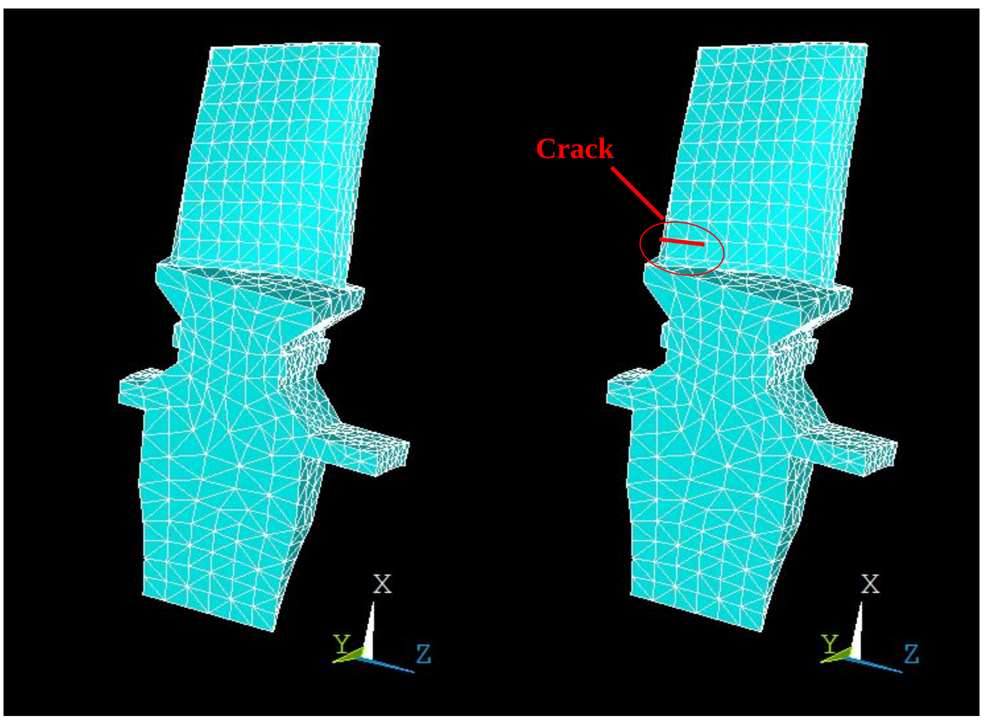

Figure 3: Geometry of single pristine sector and single cracked sector

Typically, the cracks of bladed disks are assumed permanently open and the contact between the cracked surfaces are ignored such that linear features of the dynamical system can be efficiently employed in analysis. The linear features such as modal properties can be extracted using many linear techniques [1-2] and can be used for damage detection [3-6], sensing [7-8], or online monitoring [9-10]. However, in order to estimate the dynamics of bladed disk with cracks accurately, it is important to properly model the piecewise-linear nonlinearity caused by the contact and reduce the computational complexity as much as possible.

In general, the finite element models of complex structures such as bladed disk with cracks have a large amount of degrees of freedoms (DOFs) since they need to capture the complex geometry. Likewise, it is difficult to operate the computation due to the large size of model requires a ton of memory which will cause a large number of computational cost and time. In addition, the crack existed on the bladed disk breaks the cyclic symmetry property as well. Hence, the model reduction technique called reduced-order modeling has been introduced to reduce the size of the model that will save the computational cost and time in analyzing linear systems [11-14]. The linear reduced-order models (ROMs) can be efficiently constructed using approaches based on linear transformations [15] such as component mode synthesis (CMS) [16]. However, in order to accurately estimate the dynamics of piecewise-linear nonlinear systems, the modeling of nonlinearity is required. Consequently, many linear analysis methods developed to efficiently predict the dynamics cannot be used for the analysis. Thus, several techniques have been developed to construct ROMs for these systems. These techniques include the improved reduced system [17-18], multi-level substructure [19-20], Iron-Guyan reduction [21-22], and system equivalent reduction expansion process [23-24]. These approaches keep the localized nonlinearity as master degrees of freedom (DOFs) while reducing the other DOFs using linear modes of the system.

1.2 Focus of thesis

A few years ago, many researches had constructed the ROMs of bladed disk with cracks efficiently using X-Xr approaches [25] and Craig-Bampton component mode synthesis (CB-CMS) [16] . The X-Xr approach is a fast and accurate way to model the nonlinearity of crack along the cracked surface in the bladed disk. In addition, those researchers made suggestions of using single sector model and calculations to build the ROM of bladed disks with cracks. However, those researchers actually built the ROM through using full stage model instead of single sector model and calculations.

Therefore, the purpose of this research project is to develop a faster way to build the ROMs through using single sector model and calculations. Compared with building ROM of bladed disks with cracks using full stage model, this way only need to spend the time solving the calculation of single sector. Obviously, this approach is much more efficient than before.

However, this approach has limitation that is not work for building the ROM of the bladed disk with large cracks. For constructing the ROM for the bladed disk with a large crack in one sector, the acceleration modes will be added to accelerate the convergence of calculation.

After finishing building the linear ROMs, the generalized bilinear amplitude approximation (BAA) method is combined with the nonlinear ROMs to quickly estimate the response of the piecewise-linear nonlinear system using linear analysis.

1.3 Research Objectives

The model used in the research project is a pristine sector initially. In order to build the ROM of bladed disk with cracks, the cracked sector is required to obtain. Therefore, the major objective of this research is:

- Create a small and large crack in the pristine sector to obtain the cracked sector.
- Build the ROM of a full stage bladed disk with X-Xr approach using single sector calculations.
- Using linear force response and frequency method to check the ROM of bladed disk with cracks.
- Apply modified BAA method to estimate the response of the ROM

The research project aims to develop a faster method to construct the ROM of bladed disk with a crack in one sector and efficiently estimate the dynamics of complex cracked structures. The new method incorporates the generalized BAA method with X-Xr method to compute the resonant frequency and amplitude for the system with a significant model reduction.

1.4 Overview of Thesis

The thesis has five chapters. Chapter 2 will discuss the methodology of constructing the reduced-order models of bladed disks with cracks and apply the generalized BAA method to estimate the response of the complex structure with crack. This chapter consists of three parts including constructing the ROM of bladed disks with a small crack and large crack in one sector and applying the generalized BAA approach on the simple cracked beam model to get the result of resonant frequency and amplitude of nonlinear structure. Chapter 3 focuses on the validation of checking the ROM is correct or not. Chapter 4 discusses the results of comparison of frequency and amplitude from ROM constructed in MATLAB and full stage model created in

ANSYS APDL. Moreover, the results of nonlinear vibration responses of cracked beam after computed by the X-Xr and BAA approach will be analyzed as well. Chapter 5 will concludes the results and summaries the key components of the research project. Chapter 6 will talk about the future work about the method of X-Xr and BAA approach applied on the bladed disks with a crack in one sector. The appendix A will be the data processing part which discusses the method of extraction of required data including mass, stiffness matrices, normal modes, constraint modes and acceleration.

Chapter 2: Methodology

In order to build the reduced-order model, a method to condense the DOFs of system will be developed through using the X-Xr approach and CB-CMS method.

2.1 Reduced order modeling of bladed disks with a small crack

In general, the equation of motion of linear structure can be expressed as:

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = F(t)$$

where M, C and K are the mass matrix, damping matrix and stiffness matrix, respectively. F is the periodic excitation force with excitation frequency ω. Typically, the periodic excitation force is considered to be harmonic, so the response of the system $is\ X(t) = xe^{i\omega t}$. Therefore, the equation of the motion linear structure can be shown as $(-\omega^2 M + j\omega \mathrm{C} + \mathrm{K})x(t) = F(t)$.

Next, the reordered DOFs in the global coordinate are expressed as

$$x = \begin{bmatrix} x_{c1} \\ x_{c2} \\ x_p \\ x_l \end{bmatrix}$$

where each partition represents a specific part of DOFs, it is shown below

- x_{c1} and x_{c2} represent the DOFs at the cracked surface. In general, x_{c1} is the DOFs of upper cracked surface and x_{c2} is the DOFs of lower cracked surface.
- x_p is the DOFs of the proximity area of the cracked surface
- x_l is the rest of the DOFs in the structure

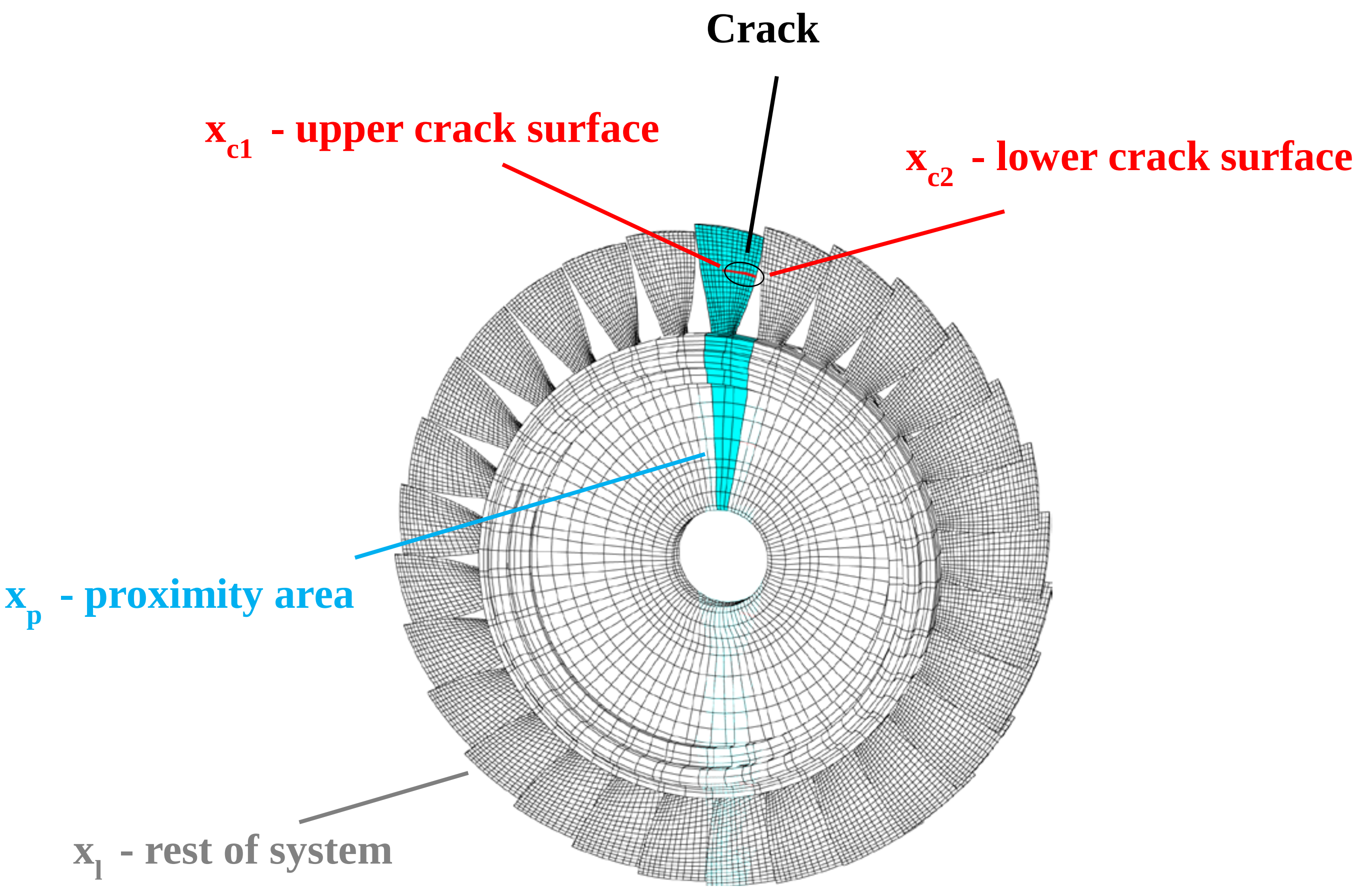

Figure 4: The reordered DOFs system

The X-Xr reduction process is introduced as follow. First, if the damping is considered as the structural damping, the general linear equation of motion of the structure can be written as follows

$$M\ddot{x}\left(t\right)+\left(1+j\gamma\right)Kx\left(t\right)=F\left(t\right)$$

where γ is the structural damping. The mass matrix and stiffness matrix can be partitioned as

$$M=\begin{bmatrix} M_{c1,c1} & M_{c1,c2} & M_{c1,p} & M_{c1,l} \\ M_{c1,c2}^{T} & M_{c2,c2} & M_{c2,p} & M_{c2,l} \\ M_{c1,p}^{T} & M_{c2,p}^{T} & M_{p,p} & M_{p,l} \\ M_{c1,l}^{T} & M_{c2,l}^{T} & M_{p,l}^{T} & M_{l,l} \end{bmatrix}$$

$$K=\begin{bmatrix} K_{c1,c1} & K_{c1,c2} & K_{c1,p} & K_{c1,l} \\ K_{c1,c2}^T & K_{c2,c2} & K_{c2,p} & K_{c2,l} \\ K_{c1,p}^T & K_{c2,p}^T & K_{p,p} & K_{p,l} \\ K_{c1,l}^T & K_{c2,l}^T & K_{p,l}^T & K_{l,l} \end{bmatrix}$$

In order to describe the relative motion of the contact surfaces, a coordinate transformation is performed as follow

$$x=\begin{bmatrix} x_{c1} \\ x_{c2} \\ x_p \\ x_l \end{bmatrix}=\alpha\begin{bmatrix} x_r \\ x_{c2} \\ x_p \\ x_l \end{bmatrix}=\alpha\begin{bmatrix} x_r \\ x_h \end{bmatrix}=\alpha\bar{x}$$

$$x_h=\begin{bmatrix} x_{c2} \\ x_p \\ x_l \end{bmatrix}$$

Where $x_r = x_{c1} - x_{c2}$ represents the relative displacement of the contact pairs. In addition, x_h represents the DOFs of the structure where the crack does not exist [25]. The transformation matrix α is the key point to transform the DOFs in the global coordinate to the relative coordinate system. The transformation matrix α is defined as

$$\alpha=\begin{bmatrix} I & I & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix}$$

After applying the transformation matrix to equation of motion of structure, a new set of equation of motion can be obtained

$$\bar{M}\ddot{\bar{x}}(t)+(1+j\gamma)\bar{K}\bar{x}(t)=\bar{F}(t)$$

Where

$$\bar{M} = \alpha^T M \alpha = \begin{bmatrix} \bar{M}_{x_r} & \bar{M}_{x_r,x_h} \\ \bar{M}^T_{x_r,x_h} & \bar{M}_{x_h} \end{bmatrix}$$

$$\bar{K} = \alpha^T K \alpha = \begin{bmatrix} \bar{K}_{x_r} & \bar{K}_{x_r,x_h} \\ \bar{K}^T_{x_r,x_h} & \bar{K}_{x_h} \end{bmatrix}$$

$$\bar{F} = \alpha^T F$$

Specific expressions for submatrices in above equation can be expressed as

$$\bar{M}_{x_r} = M_{c1,c1}$$

$$\bar{K}_{x_r} = K_{c1,c1}$$

$$\bar{M}_{x_r,x_h} = \begin{bmatrix} M_{c1,c1} + M_{c1,c2} & M_{c1,p} & 0 \end{bmatrix}$$

$$\bar{K}_{x_r,x_h} = \begin{bmatrix} K_{c1,c1} + K_{c1,c2} & K_{c1,p} & 0 \end{bmatrix}$$

$$\bar{M}_{x_h} = \begin{bmatrix} M_{c1,c1} + M^T_{c1,c2} + M_{c1,c2} + M_{c2,c2} & M_{c1,p} + M_{c2,p} & 0 \\ M^T_{c1,p} + M^T_{c2,p} & M_{p,p} & M_{p,l} \\ 0 & M^T_{p,l} & M_{l,l} \end{bmatrix}$$

$$\bar{K}_{x_h} = \begin{bmatrix} K_{c1,c1} + K^T_{c1,c2} + K_{c1,c2} + K_{c2,c2} & K_{c1,p} + K_{c2,p} & 0 \\ K^T_{c1,p} + K^T_{c2,p} & K_{p,p} & K_{p,l} \\ 0 & K^T_{p,l} & K_{l,l} \end{bmatrix}$$

In these submatrices, one key point is that matrices $M_{c1,l}$, $M_{c2,l}$, $K_{c1,l}$, $K_{c2,l}$ are all zero matrices because there is no relations between the DOFs of cracked surface and the rest of DOFs in the structure.

Next, the CB-CMS approach is employed to reduce the model size where by the relative DOFs grouped in x_r are chosen to be active while all other DOFs in x_h are reduced using normal modes $\boldsymbol{\phi}_n$. The Craig-Bampton transformation matrix β can be expressed as

$$\beta = \begin{bmatrix} I & 0 \\ \psi_c & \phi_n \end{bmatrix}$$

Where Ψ_C is the matrix of constraint modes and $\boldsymbol{\phi}_n$ is the matrix of normal modes. The constraint modes are defined as the static displacement of the DOFs in x_h due to successive unit displacement of each relative DOFs in x_r, while keeping all other active DOFs fixed. Namely, the constraint modes are nonzero approximately in the cracked sector except the boundary of DOFs. In other words, the constraint modes of other pristine sectors are zero definitely. The normal modes can be obtained by solving for the eigenvalue problem for the corresponding pristine structure using cyclic symmetry. Furthermore, these normal modes can be truncated for a frequency range of interest to reduce the model size. That means both constraint modes and normal modes can be expressed through using single cracked sector and pristine sector.

Reduced coordinates can be obtained by applying the Craig-Bampton transformation

$$\bar{x} = \begin{bmatrix} x_r \\ x_h \end{bmatrix} = \beta \begin{bmatrix} x_r \\ \eta \end{bmatrix} = \beta \tilde{x}$$

Where the size of x_r is the number of constraint modes included in Ψ_C and the size of η is the number of normal modes included in $\boldsymbol{\phi}_n$.

In addition, the equations of motion can be reduced to

$$M_{ROM} \ddot{\tilde{x}}(t) + (1 + j\gamma) K_{ROM} \tilde{x}(t) = F_{ROM}(t)$$

Where

$$M_{ROM} = \beta^T \bar{M} \beta = \begin{bmatrix} M_{x_r} & M_{x_r,\eta} \\ M_{x_r,\eta}^T & M_\eta \end{bmatrix}$$

$$K_{ROM} = \beta^T \bar{K} \beta = \begin{bmatrix} K_{x_r} & K_{x_r,\eta} \\ K_{x_r,\eta}^T & K_\eta \end{bmatrix}$$

$$F_{ROM} = \beta^T \bar{F}$$

The submatrices in above equation can be expressed as

$$M_{x_r} = \bar{M}_{x_r} + \psi_c^T \bar{M}_{x_r,x_h}^T + \bar{M}_{x_r,x_h} \psi_c + \psi_c^T \bar{M}_{x_h} \psi_c$$

$$K_{x_r} = \bar{K}_{x_r} + \psi_c^T \bar{K}_{x_r,x_h}^T + \bar{K}_{x_r,x_h} \psi_c + \psi_c^T \bar{K}_{x_h} \psi_c$$

$$M_{x_r,\eta} = \bar{M}_{x_r,x_h} \phi_n + \psi_c^T \bar{M}_{x_h} \phi_n$$

$$K_{x_r,\eta} = \bar{K}_{x_r,x_h} \phi_n + \psi_c^T \bar{K}_{x_h} \phi_n$$

$$M_\eta = \phi_n^T \bar{M}_{x_h} \phi_n = I$$

$$K_\eta = \phi_n^T \bar{K}_{x_h} \phi_n = \Lambda$$

Note that M_η is the identity matrix and K_η is the eigenvalue matrix of the pristine structure. This two matrices can be computed to check the ROMs is correct or not simply. Most importantly, Ψ_C $\boldsymbol{\phi}_n$,M and K matrices can be constructed in terms of single sector quantities. It decreases the computational time and cost of building the ROM a lot.

2.2 Reduced order modeling of bladed disks with a large crack

For the case of building the ROMs for bladed disks with a large crack in one sector, the method discussed above will spend a lot of time to obtain the convergence solution. Because the normal modes will require a large amount of modes of mode shapes to make the calculation converge. Moreover, the process of convergence will waste a lot of time on it. Therefore, the acceleration modes will be added to the Craig-Bampton transformation matrix β to generate the modified Craig-Bampton transformation matrix β_m to accelerate the convergence of calculation.

The modified Craig-Bampton transformation matrix β_m is defined as

$$\beta_m = \begin{bmatrix} I & \phi^r_{CBc} & 0 \\ \psi_c & \phi_{CBc} & \phi_n \end{bmatrix}$$

Where $\phi^r_{CBc} = \phi_{CBc_{c1}} - \phi_{CBc_{c2}}$ represents mode shape of cracked surface in the relative coordinate. The expansion of ϕ_{CBc} is $\phi_{CBc} = [\phi^T_{CBc_{c2}}\ \phi^T_{CBc_p}\ \phi^T_{CBc_l}]^T$. Note that the acceleration mode are nonzero only in the cracked sector except the boundary of DOFs.

The ROM of bladed disks with a large crack in one sector can be obtained by applying the modified Craig-Bampton transformation matrix β_m. Other steps are completely same with the method discussed above.

2.3 Bilinear amplitude approximation (BAA) to estimate the resonant frequency and amplitude

A new method to approximate resonant frequency and amplitude for nonlinear structures called generalized bilinear amplitude approximation (BAA) is proposed in this section.

Typically, an elastic structure involving intermittent contact experiences two linear states, which are the open state, and sliding state. The system will switch between these two linear states. Thus, one entire vibration cycle can be approximated by these two states. Figure 5 is the one steady-state vibration cycle.

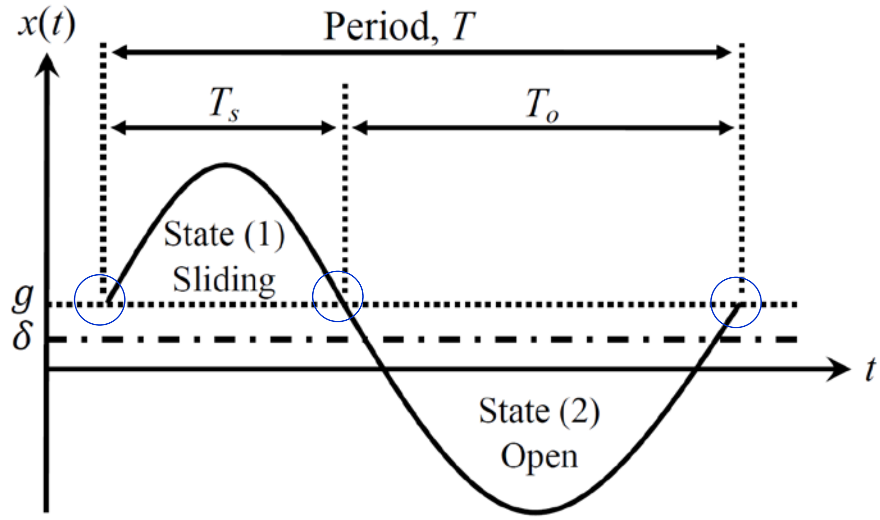

Figure 5: One steady-state vibration cycle [26]

Where T_s is the time portion that system spends in state 1, T_o is the time portion that system spends in state 2 and T is the total time period of periodic response which is the summation of T_s and T_o.

In the modal coordinate, the equations of the response of nonlinear structure in the sliding states and open states are shown below

$$\tilde{q}_{s,i}(t) = e^{-\zeta_{s,i}\omega_{s,i}t}[s_{1,i}\cos(\omega_{sd,i}t) + s_{2,i}\sin(\omega_{sd,i}t)] + \frac{(f_{s,i}/\omega_{s,i}^2)\cos(\omega t - \theta_{s,i} + \alpha)}{\sqrt{[1-(\omega/\omega_{s,i})^2]^2 + (2\zeta_{s,i}\omega/\omega_{s,i})^2}}$$

$$q_{o,j}(t) = e^{-\zeta_{o,j}\omega_{o,j}t}[o_{1,j}\cos(\omega_{od,j}t) + o_{2,j}\sin(\omega_{od,j}t)] + \frac{(f_{o,j}/\omega_{o,j}^2)\cos(\omega t - \theta_{o,j} + \alpha)}{\sqrt{[1-(\omega/\omega_{o,j})^2]^2 + (2\zeta_{o,j}\omega/\omega_{o,j})^2}}$$

Where subscript s represents sliding states, subscript o is open states. α is the phase difference between the excitation and the piecewise linear response. And n_s and n_o are the numbers of modes used to project the motion in state 1 and state 2, respectively. The point marked by blue circle is the boundary point.

In this two equations, the unknown parameters are α, o_1, o_2, s_1 and s_2. And then, the boundary conditions that is marked by blue circle are applied to solve these unknown parameters to obtain the response of the nonlinear structure.

Chapter 3: Validation

The validation chapter is required to check the constructed ROMs is correct or not. In the vibration analysis, two key components that are frequency and amplitude of the system will be focused.

3.1 Validate the linear natural frequency of the ROM

In the validation of linear natural frequency of the system, the main idea is to compare the frequency of the model of bladed disks with a crack in one sector obtained in ANSYS APDL and the linear natural frequency of ROMs obtained in MATLAB.

The frequency of full stage model in ANSYS APDL will be extracted through using the ANSYS APDL command to build the model of bladed disks with a crack in one sector called full stage model first and apply the boundary condition to run the modal analysis. Therefore, the frequency of the full stage model in ANSYS APDL is easy to obtain. For the linear natural frequency of constructed ROMs, it can exploit the MATLAB command (EIG) to run the modal analysis to get the frequency based on the reduced order model of mass and stiffness matrix. The MATLAB command will be shown below as

$$[Eigenvector, Eigenvalue] = eig(K_{ROM}, M_{ROM})$$

Where M_{ROM} is the ROM of mass matrix and K_{ROM} is the ROM of stiffness matrix. From this MATLAB command, the eigenvalue of the system can be obtained. The eigenvalue of the system is the square of frequency of the system. Hence, the MATLAB command to get the frequency of system is expressed as

$$frequency = \frac{\sqrt{eigenvalue}}{2\pi}$$

Where the unit of the frequency is Hz.

Definitely, the frequency of full stage model in ANSYS APDL should be same with the linear natural frequency of constructed ROMs. However, small error will happen in the process of calculation or transformation. Therefore, the frequency deviation should be computed to look at the error. The formula of frequency deviation is expressed as

$$deviation = \left| \frac{\omega_{ROM} - \omega_a}{\omega_a} \right| * 100\%$$

Where ω_{ROM} is the frequency of constructed ROM in MATLAB and ω_a is the exact frequency of the full stage model in ANSYS APDL.

3.2 Validate the linear force response of the ROM

In the validation of amplitude of the system, the main idea is to compare the amplitude of the full stage model obtained in ANSYS APDL and the linear amplitude of ROMs obtained in MATLAB.

The amplitude of the full stage model can be computed through using ANSYS APDL command followed by three steps including building the cracked model, applying the force to the model and running the force response analysis. For the linear amplitude of constructed ROMs, it can be calculated based on applying the force to the model and running the force response analysis. The formula to compute the amplitude of constructed linear ROMs is expressed as

$$Q = (-\omega^2 M_{ROM} + (1 + j\gamma) K_{ROM})^{-1} F_{ROM}$$

Where ω is excitation frequency, M_{ROM} is the ROM of mass matrix, K_{ROM} is the ROM of stiffness matrix, F_{ROM} is the ROM of force and γ is the structural damping. The method to obtain the ROM of force has been discussed in the chapter 2.1.

To be precise, the amplitude of the full stage model in ANSYS APDL ought to be same with the linear amplitude of constructed ROMs. However, the small error during the process of calculation is unavoidable.

Chapter 4: Results

4.1 ROM of bladed disks with a small crack in one sector

The ROM of bladed disk with a small crack in one sector has been built successfully. There are 6 constraint modes, 234 normal modes are used to construct the β matrix. Hence, the ROM has 240 DOFs after the CB-CMS reduction. The crack is located at the root of the trailing edge of the blade. The length of crack is almost one-fifth of the blade chord length. The crack consists 2 contact pairs that means there are 6 DOFs along the crack surface.

Figure 6 is the linear excitation frequency of FEM and ROM for small crack. For this case, first two-mode family is the interested range. From the figure 6, the two curves of FEM and ROM almost matches well. Namely, the constructed ROM is correct based on the linear frequency checking. Note that the linear natural frequency of FEM is the exact solution.

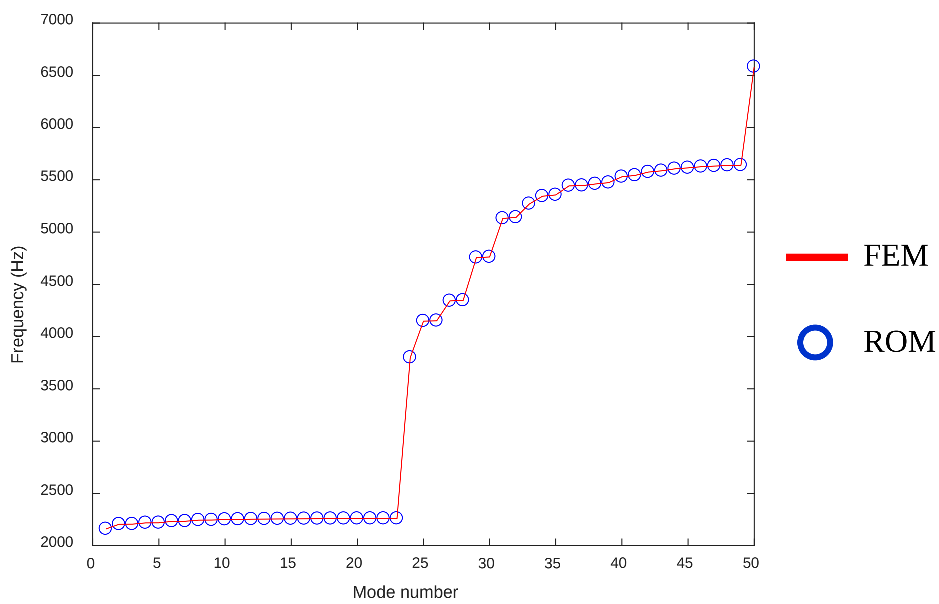

Figure 6: Linear excitation frequency of FEM and ROM (small crack)

Thus, the frequency deviation need to be plot to look at the relative error between ROMs and FEM. Figure 7 is the frequency deviation of ROM for small crack. Obviously, the maximum relative error percent is almost 0.043%. Namely, the relative error of the interested range are all below 0.045%.

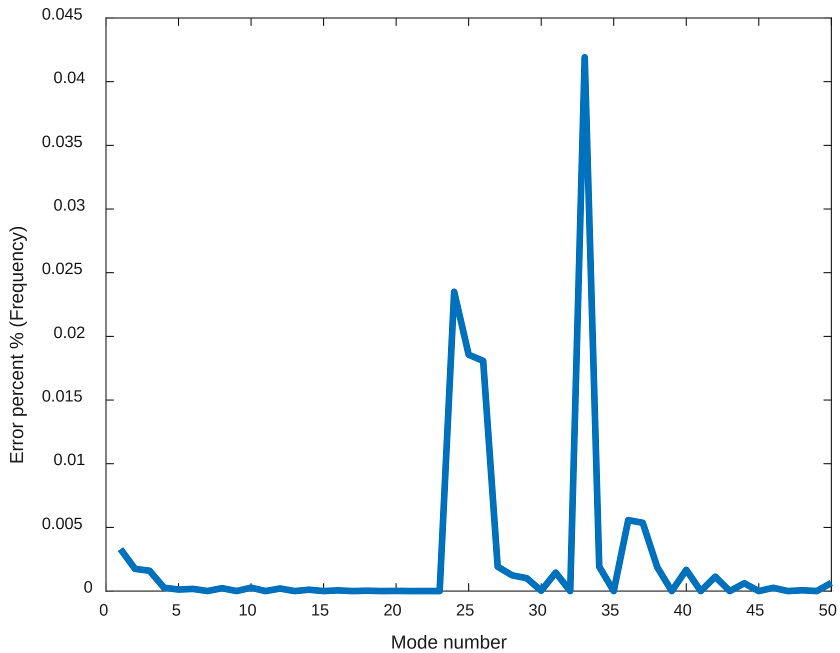

Figure 7: The frequency deviation of ROM (small crack)

In the amplitude checking, the force applied to the tip node z direction at trailing edge of cracked sector is 100000 N and the engine excitation order is one. The structural damping ratio is $1.5 * 10^{-7}$. Figure 8 is linear amplitude of FEM and ROM for small crack. Note that the amplitude of FEM is the exact solution.

From the figure 8, the ROM results match with FEM results very well obviously. Namely, the constructed ROM of bladed disk with a small crack in one sector is correct based on the frequency and amplitude checking through using the linear analysis.

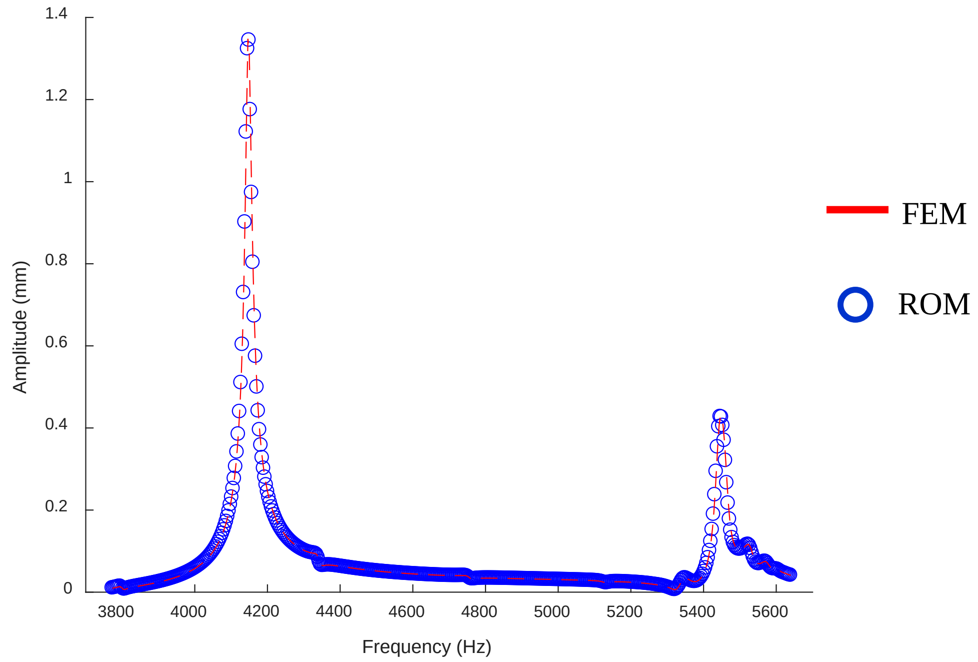

Figure 8: Linear amplitude of FEM and ROM (small crack)

4.2 ROM of bladed disks with a large crack in one sector

The ROM of bladed disk with a large crack in one sector has been built successfully. There are 18 constraint modes, 234 normal modes and 15 acceleration modes are used to construct the β_m matrix. Hence, the ROM has 267 DOFs after the CB-CMS reduction. The crack is located at the root of the trailing edge of the blade. The length of crack is almost 50% of the blade chord length. The crack consists 6 contact pairs that means there are 18 DOFs along the crack surface.

Figure 9 is the linear frequency plot of ROM and FEM for large crack. For this case, first two-mode family is the interested range. The linear frequency plot of ROM constructed in MATLAB looks pretty match with the linear frequency plot of FEM constructed in ANSYS.

That means the built ROMs is correct based on the frequency checking. Note that the frequency of FEM in ANSYS is the exact solution.

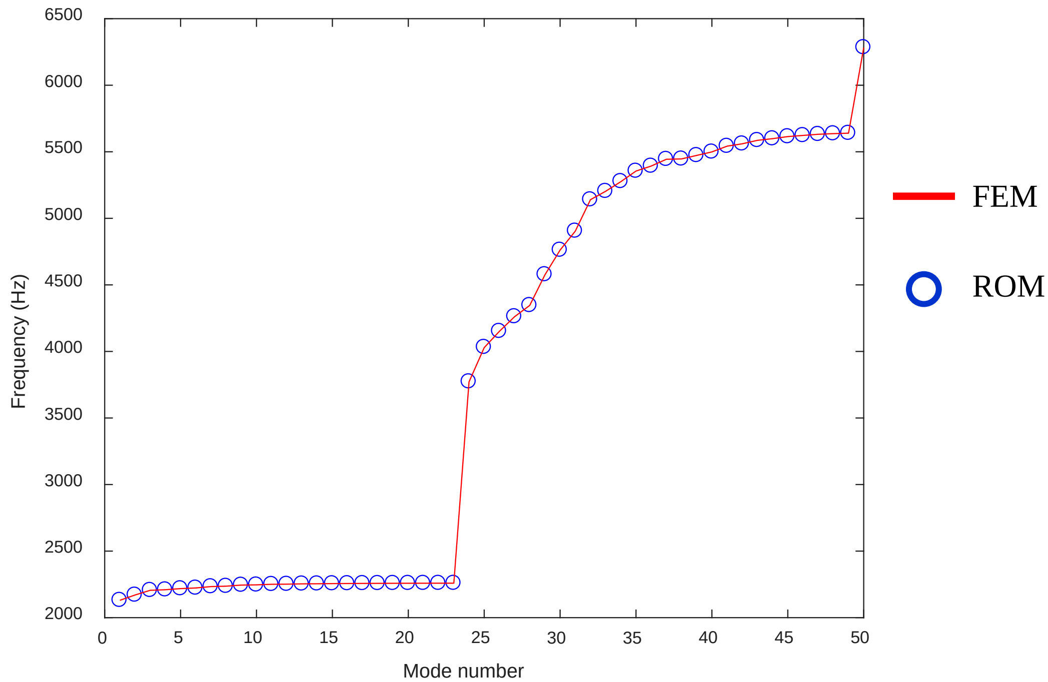

Figure 9: Linear excitation frequency of FEM and ROM (large crack)

Thus, the frequency deviation need to be plot to look at the relative error between ROMs and FEM. Figure 10 is the frequency deviation plot of ROM for large crack. Obviously, the maximum relative error percent is almost 0.075%. Namely, the relative error of the interested range are all below 0.08%.

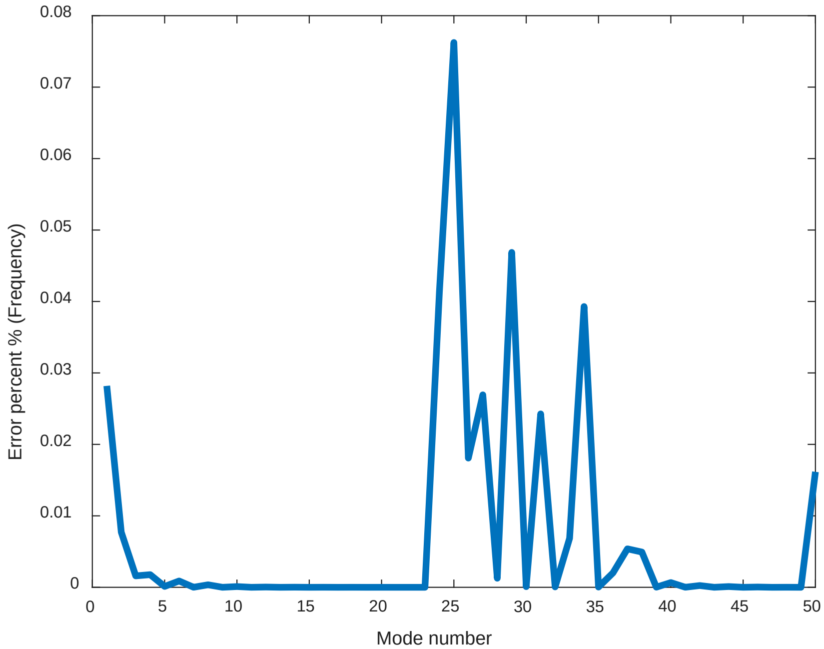

Figure 10: The frequency deviation of ROM (large crack)

In the amplitude checking, the force applied to the tip node z direction at trailing edge of cracked sector is 100000 N and the engine order is one. The structural damping ratio is $1.5 * 10^{-7}$ as well. Figure 11 is the plot of linear amplitude of FEM and ROM for large crack. Note that the amplitude of FEM in ANSYS is the exact solution. From the comparison plot, the amplitude plot of ROM matches with the amplitude plot of FEM very well. Hence, the constructed ROM is correct based on the frequency and amplitude checking through using linear analysis.

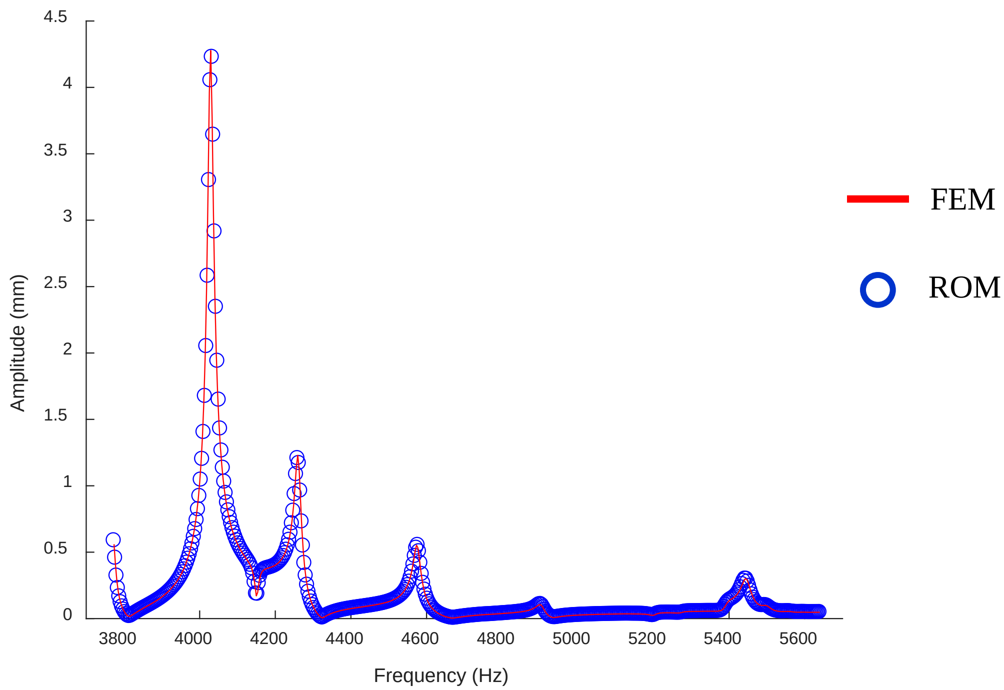

Figure 11: Linear amplitude of FEM and ROM (large crack)

4.3 Comparison of FEM and ROM with small and large crack

Compared with the number of DOFs used in the FEM, the number of DOFs used in ROM is much smaller relatively. The table of number of DOFs used in the FEM and ROM is shown below

	FEM	**ROM with small crack**	**ROM with large crack**
Number of DOFs	**86000**	**240**	**267**

Table 1: Number of DOFs in FEM and ROM

Obviously, the number of DOFs from FEM to ROM will have a huge reduction. Namely, the reduction technique called X-Xr approach is to condense the size of the system successfully.

4.4 Cracked beam model

The cracked cantilevered beam [26] studied in this work is schematically shown in Figure 12 and the corresponding finite element (FE) model is shown in Figure 13. The beam is excited by a harmonic force $F(t) = F_0 \cos(\omega t)$, where $F_0 = 1\ N$, in the y direction at the tip. Furthermore, the gap/prestress will exist when a pair of static forces F_{st} are applied in the x-direction. The crack is located at L_C = 0.09 m which is one-eighth of the length L from the fixed end of the beam and has a depth of H_c = 0.016 m which is 50% of the Height H. The crack is spread across the entire width. Structural damping $C = \gamma K$, where $\gamma = 5 * 10^{-4}$, is used in the model.

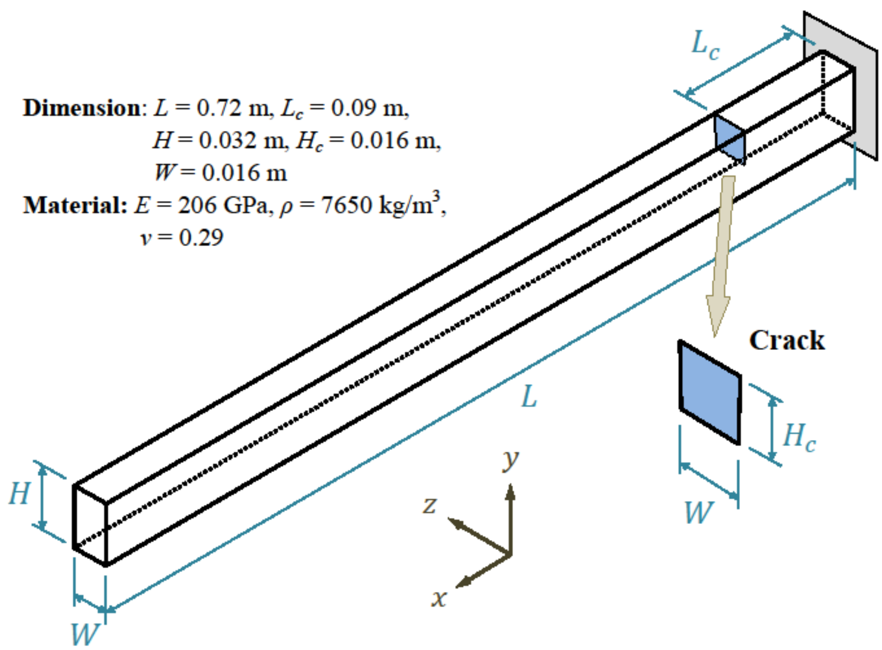

Figure 12: Schematic plot of the cracked beam used in this work

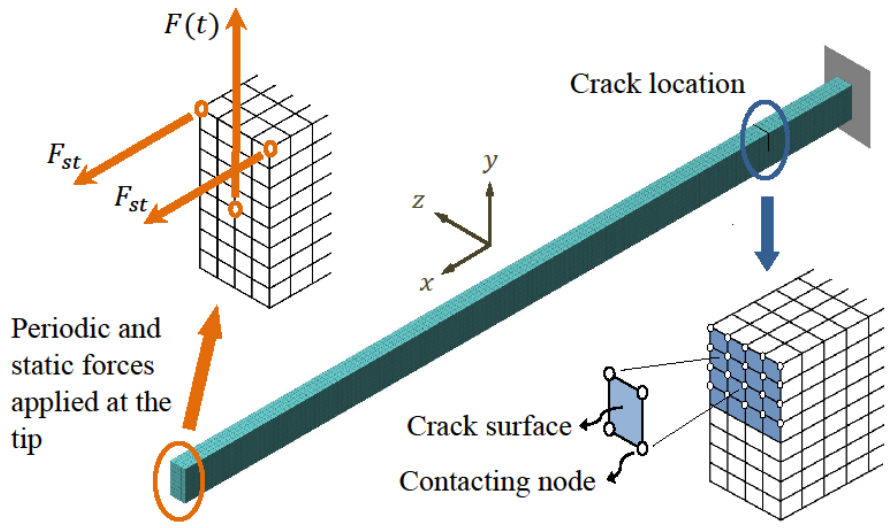

Figure 13: Finite element model of the beam and periodic and static forces applied at the free end

4.4.1 Model reduction

The total DOFs of the original FE model is 19500 including 120 DOFs on the crack surfaces. The 120 DOFs includes 40 contacting nodes and 20 contact pairs. In order to obtain an effective ROM, 60 constraint modes and 300 normal modes are used to construct the β matrix. The ROM has 360 DOFs after the CB-CMS reduction. In order to validate the dynamic characteristics of the ROM, the linear natural frequencies of the ROM and original FE model (the system always stays in its open state and interpenetration is allowed between the contact surfaces) are compared and plotted in Figure 14. The relative errors of the natural frequencies of ROM are all less than 0.02% over the plotted mode number range as shown in Figure 15.

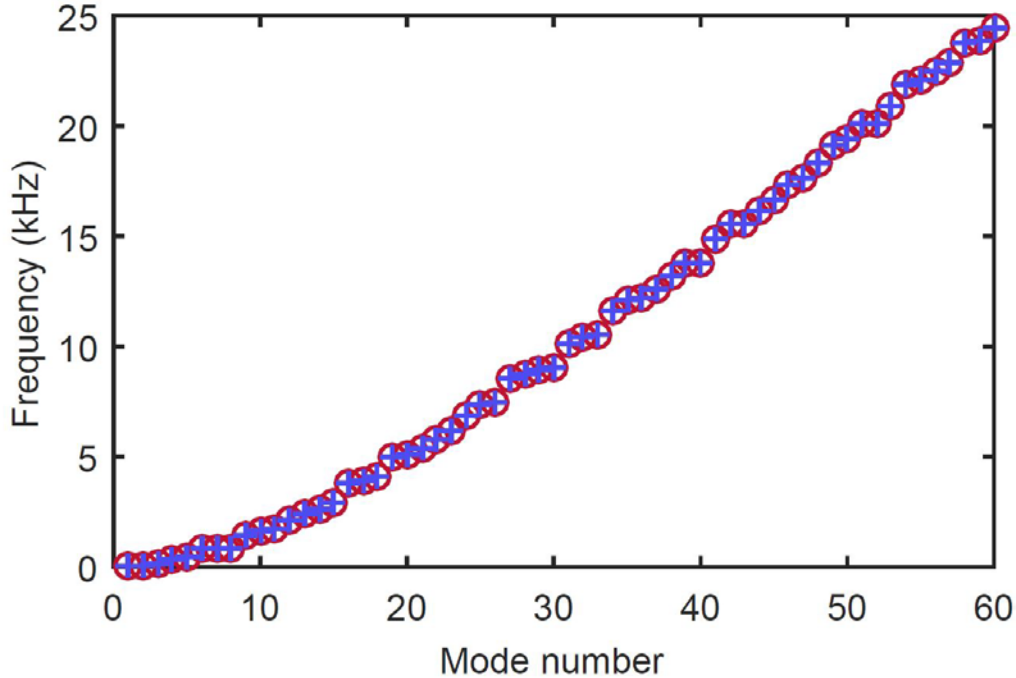

Figure 14: The linear natural frequencies of the ROM and the FE model (+ for the ROM and o for the original FE model)

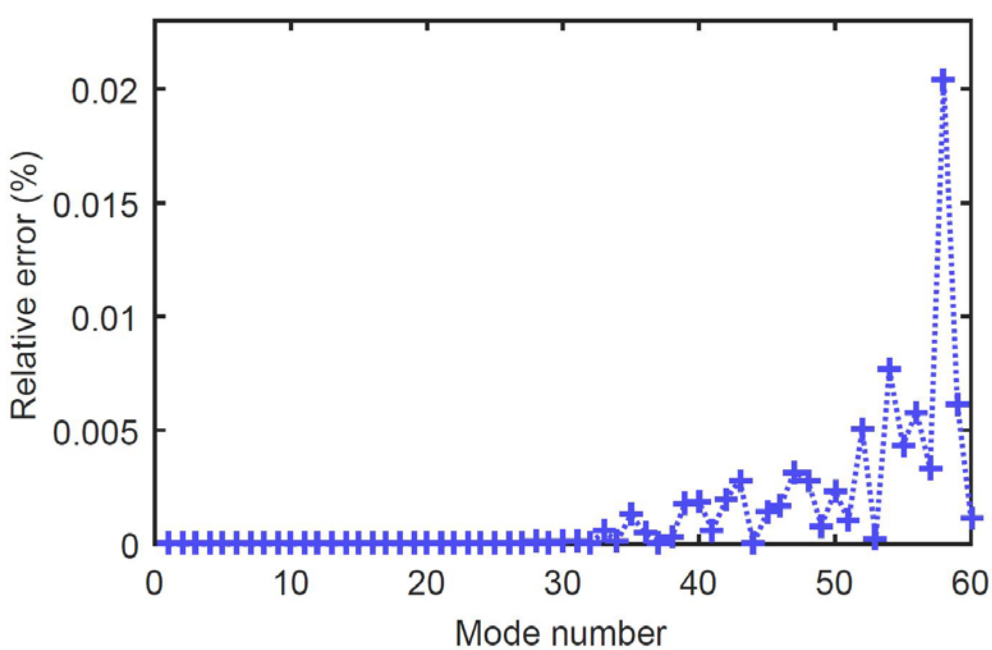

Figure 15: The relative errors of the linear natural frequencies of the ROM

4.4.2 Response approximation

The first in-plane bending modes (x-y plane) for the sliding and open system were selected to construct the normal modes of open and sliding state. The contact stiffness $k^*= 5 * 10^{11}$ N/m and damping $C^* = 2.5 * 10^8$ kg/s, which are at least two orders of magnitude greater than the structural stiffness and damping, were used for this system. Comparison of the approximate amplitude at the free end computed by the X-Xr-BAA method and the generalized BAA method, and the exact amplitude obtained by time integration for three cases are shown in case Figure 16, respectively. The approximated amplitude computed by the generalized BAA was based upon the original FE model of the beam without the X-Xr reduction process [26]. The exact solution was computed by the Newmark method [28] and was performed using the commercial software ANSYS Workbench.

The approximated amplitude obtained by the X-Xr-BAA method at the resonant frequency is $10.15 * 10^{-5}\ m$, respectively. The exact amplitude computed by time integration is $10.30 * 10^{-5}\ m$, respectively. The relative error of the approximated amplitude computed by the X-Xr-BAA method at the resonant frequency is 1.46%. The approximated response of the ROM reaches the same accuracy as the approximated response of the original FE model and agrees well with the exact response over the plotted frequency range.

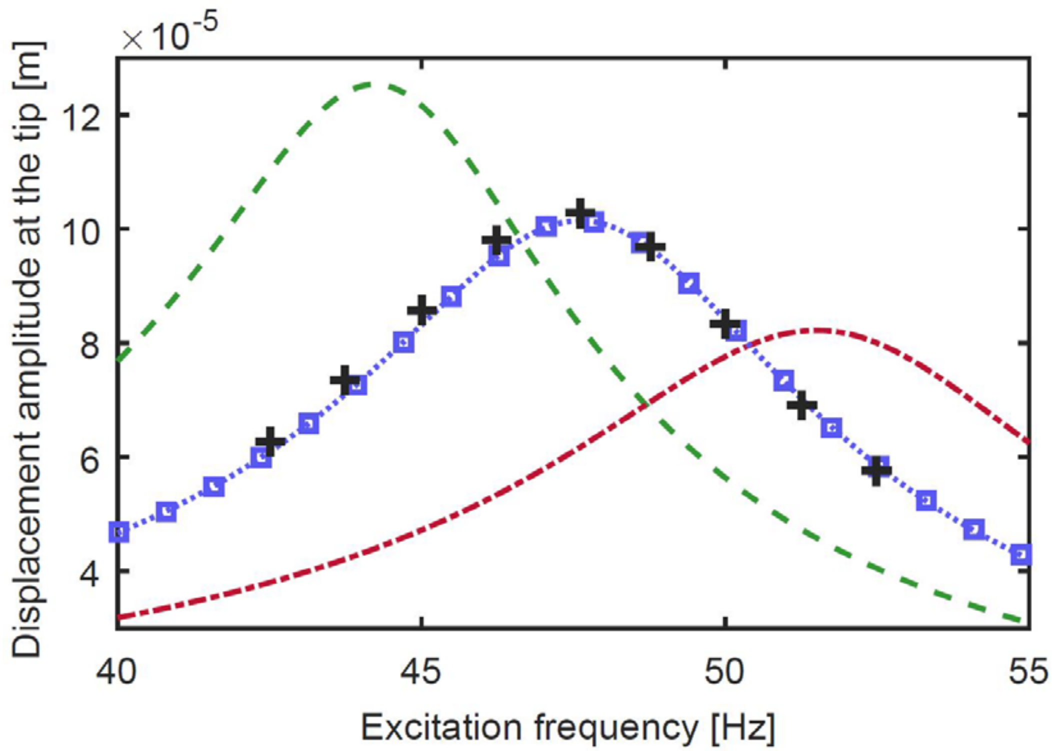

Figure 16: Nonlinear vibration response for system

Where X-Xr-BAA (□), the generalized BAA without model reduction (···) and time integration (+). The linear vibration responses of the system in the sliding state is denoted by (—·). The linear vibration responses of the system in the open state is denoted by (---).

Chapter 5: Conclusion

An efficient methodology for constructing the ROM of bladed disks with a crack in one sector through using X-Xr approach and CB-CMS method based on the sector level quantities was developed. Thus, this reduction technique was combined with a bilinear amplitude approximation (BAA) approach to predict the vibration response of the nonlinear structure using linear analysis. Moreover, the BAA method was based on the idea that a nonlinear structure that is driven by harmonic forces will switch between two linear states, which are referred to as the open, and the sliding states. The reduced-order models (ROMs) for the two linear systems can be obtained using the reduction technique developed in this thesis and the response of the nonlinear structure including the resonant frequency and amplitude can be approximated using the BAA method. Consequently, a significant computational savings can be achieved.

Chapter 6: Future work

In the future, the X-Xr-BAA approach will be exploited to estimate the response of complex structure including the blade disks with a crack in one sector. And investigate influences of mistuning, crack location and crack size on the dynamics of bladed disks.

Appendix A: Data Processing

The method of obtaining the data including mass, stiffness matrices, normal modes, constraint modes and acceleration modes will be discussed through using the multiple software including ANSYS APDL and MATLAB.

A.1 Extract the mass and stiffness matrices

In order to build the ROMs of bladed disks with small or large cracks, the mass and stiffness matrices are required. The required data mentioned above is the single sector of mass and stiffness matrices including the pristine sector and cracked sector.

The approach to extract the mass and stiffness matrices are to fix the boundary condition and run the analysis to obtain through using HBMAT command in the ANSYS APDL. The mass and stiffness matrices extracted from ANSYS APDL is HB format in the cylindrical coordinate system. However, HB format cannot be imported into MATLAB directly. The HB format has to be converted to MATLAB format. Followed by the method of building ROMs discussed above, the mass and stiffness matrices of cracked sector has to be reordered the DOFs. Afterwards, reordered mass, stiffness matrices of cracked sector combines with the mass, stiffness matrices of pristine sector to generate the mass, stiffness matrices of full stage model that is M and K matrix shown in the chapter 2.1 in the cylindrical coordinate system.

A.2 Extract the normal modes

In the Craig-Bampton transformation matrix β, the normal modes can be used to capture the motion of pristine structure of bladed disks. The normal modes are computed by single pristine sector through using cyclic command to run the modal analysis in the ANSYS APDL. During the process of computation, ANSYS APDL will create the duplicate sector to help calculation. Note that the normal modes extracted from the ANSYS APDL is cyclic sector mode shape in the cylindrical coordinate system. The expanded equation will be used to expand modes pairs from cyclic sectors to full expanded coordinates in the cylindrical coordinate system.

The expanded formula is shown below as

$$\phi_j^k = \phi_A^k \sqrt{\frac{2}{N}} cos[\frac{2\pi}{N} k(j-1)] - \phi_B^k \sqrt{\frac{2}{N}} sin[\frac{2\pi}{N} k(j-1)]$$

Where k is harmonic number, j is sector number, N is number of sector. For this case, N is 23 that means the turbomachinery has 23 sectors. A is cyclic sector mode shape for original sector. B is cyclic sector mode shape for duplicate sector. The ϕ matrix calculated by expanded formula is the matrix $\boldsymbol{\phi}_n$ in the Craig-Bampton transformation matrix β.

A.3 Extract the constraint modes

In the Craig-Bampton transformation matrix β, the constraint modes can capture motions of cracked sector not contained within the pristine system. The constraint modes can be computed by single cracked sector based on the formula [16], which is shown below as

$$[\psi_c] = -[K^{II}]^{-1}[K^{IB}]$$

Where Ψ_C is the constraint modes of single cracked sector, K is the stiffness matrix. The superscript B and I refer to boundary and interior. For this case, the boundary is the DOFs in the relative coordinate (x_r) and interior is the DOFs of crack sector (x) except the boundary condition. Hence, the constraint modes of single cracked sector can be obtained. Due to the constraint modes of other pristine sector are all zero, the constraint modes of full stage model can be generated easily.

A.4 Extract the acceleration modes

For the acceleration modes, it is the mode shape of the single cracked sector. The acceleration modes aim to accelerate the convergence of calculation. Namely, the acceleration modes can save a large amount of computational cost.

Therefore, the acceleration modes of cracked sector is convenient to be computed through applying the boundary condition to the model and running the modal analysis to obtain it. The acceleration modes of full stage model can be generated by the acceleration modes of cracked sector and other pristine sectors. Note that the acceleration modes of other pristine sectors are approximately zero. Therefore, the acceleration modes of full stage model can be constructed easily as well.

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